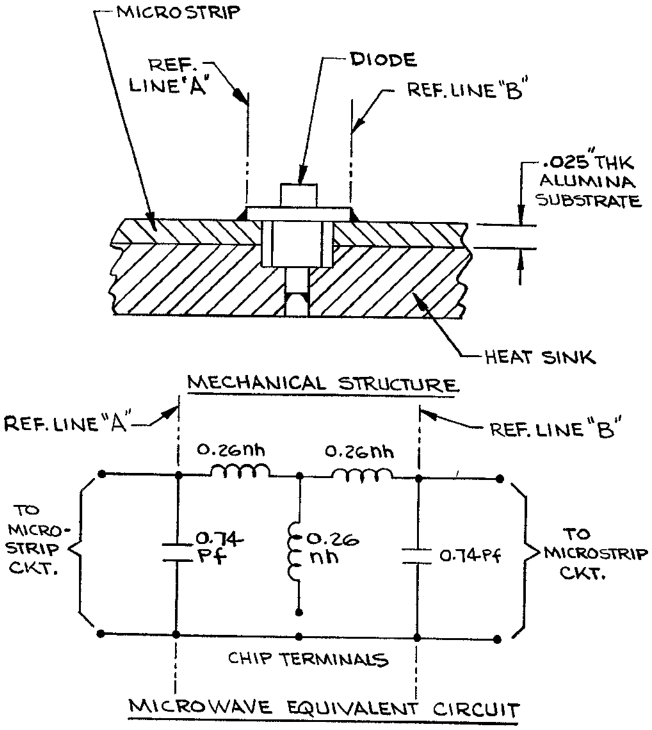

Fig. 3.

The matrices for the low-pass pi-network for the empty package can be represented by the matrix multiplication of the cascaded unit cells

$$T_\pi = \begin{vmatrix} 1 & 0 \\ jB_a & 1 \end{vmatrix} \begin{vmatrix} 1 & \frac{1}{jB_b} \\ 0 & 1 \end{vmatrix} \begin{vmatrix} 1 & 0 \\ jB_a & 1 \end{vmatrix} = \begin{vmatrix} 1 + B_a/B_b & -j/B_b \\ jB_a(2 + B_a/B_b) & 1 + B_a/B_b \end{vmatrix} \tag{2}$$

where B_a and B_b are the susceptances of C_A and L_B, respectively.

The matrix for the transmission line can be represented by

$$T_L = \begin{vmatrix} \cos\theta & jz_0 \sin\theta \\ jY_0 \sin\theta & \cos\theta \end{vmatrix} \tag{3}$$

where

$$\theta = 2\pi \frac{x}{\lambda_m}, \qquad x = 0.336 \text{ in}$$

$$Y_0 = 1/z_0, \qquad z_0 = 50\,\Omega.$$

By repeated application of matrix multiplication, the equivalent circuit for the empty package and the long resonator can be represented by the $ABCD$ matrix

$$|T_L|\,|T_\pi|\,|T_L| = \begin{vmatrix} A & B \\ C & D \end{vmatrix}. \tag{4}$$

The input admittance is

$$Y = \frac{AG_L + B}{CG_L + D} \tag{5}$$

where G_L is the load conductance of the open-circuited transmission line and was taken to be 0.001 mho. When $Y=0$, parallel resonance is achieved.

A digital computer using the Monsanto Company's Microwave Circuit Analysis program was used to determine the element values for the diode package. The procedure consists of selecting values for C_A and L_B, then calculating the $ABCD$ matrix, equation (4), and then solving for the admittance Y equal to zero. An error function was used to minimize the frequency differences between the measured and the calculated resonant frequencies. A subroutine prints out the calculated resonant frequencies and the error function. The optimized values for L_B and C_A were 0.52 nH and 0.74 pF, respectively. The experimental and computed circuit model resonances are tabulated in Table I (b).

The empty package was replaced with an identical package in which a 6-mil ribbon lead was connected from the top hat to the pedestal at the base of the package. With the addition of the inductance L_C, the resonances again shifted frequency and a new resonant frequency occured. A matrix similar to the open-circuited package case with the same values for L_B and C_A was used to obtain the optimum lead inductance L_C. The computed value was 0.26 nH. Both the experimental and computed resonances are shown in Table I (c).

In conclusion, a very simple lumped circuit gave fairly good agreement. Better results could have been achieved if the parameters of the pi-network were not invariant with change in measured frequency. The transformation of impedances to the active chip terminals (Fig. 3) now becomes a matter of ordinary RLC network theory.

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The Impedance and Scattering Properties of a Perfectly Conducting Strip Above a Plane Surface-Wave System

EDMOND S. GILLESPIE AND FRANCIS J. KILBURG

***Abstract*—The impedance and scattering properties of a perfectly conducting strip above a dielectric-coated conducting plane is investigated both theoretically and experimentally. An integral equation for the induced current is presented and solved numerically using a point-matching technique. The values of the reflection and transmission coefficients are calculated from the computed current distributions. The results of the computations are compared to the measured values and the agreement is quite good. In addition the impedance and fractions of power reflected, radiated, and transmitted are computed and displayed graphically.**

I. Introduction

Whenever an obstacle is placed in the vicinity of an unshielded surface-wave system, part of the scattered field radiates away from the surface, part is backscattered in the form of a surface wave, and the remainder is transmitted in the forward direction also as a surface wave. The obstacle can be thought of as an antenna fed by a surface-wave transmission line [1], [2]. It is, therefore, altogether appropriate to characterize the obstacle by the usual parameters from transmission-line theory; namely, by impedance or scattering matrices. An aspect of this type of scattering problem which has been largely neglected in the literature is that of the prediction of the fraction of power lost by radiation.

Gillespie and Gustincic [3], [4] have computed the reflection coefficients of strips above a dielectric-coated conducting plane and of plane conducting annuli surrounding a Goubau line. Using results presented in [4], Gillespie [5] calculated the shunt impedance of plane annuli on a Goubau line, as well as the fractions of power radiated, reflected, and transmitted. It is the purpose of this short paper

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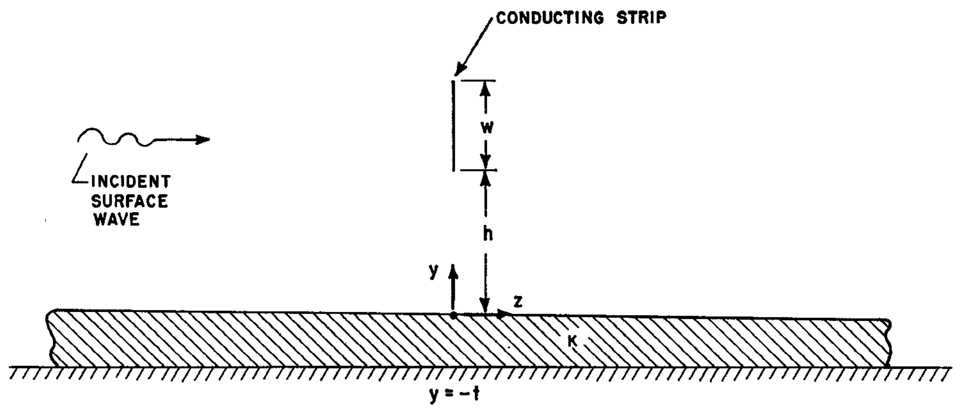

Fig. 1. Geometry of the problem.

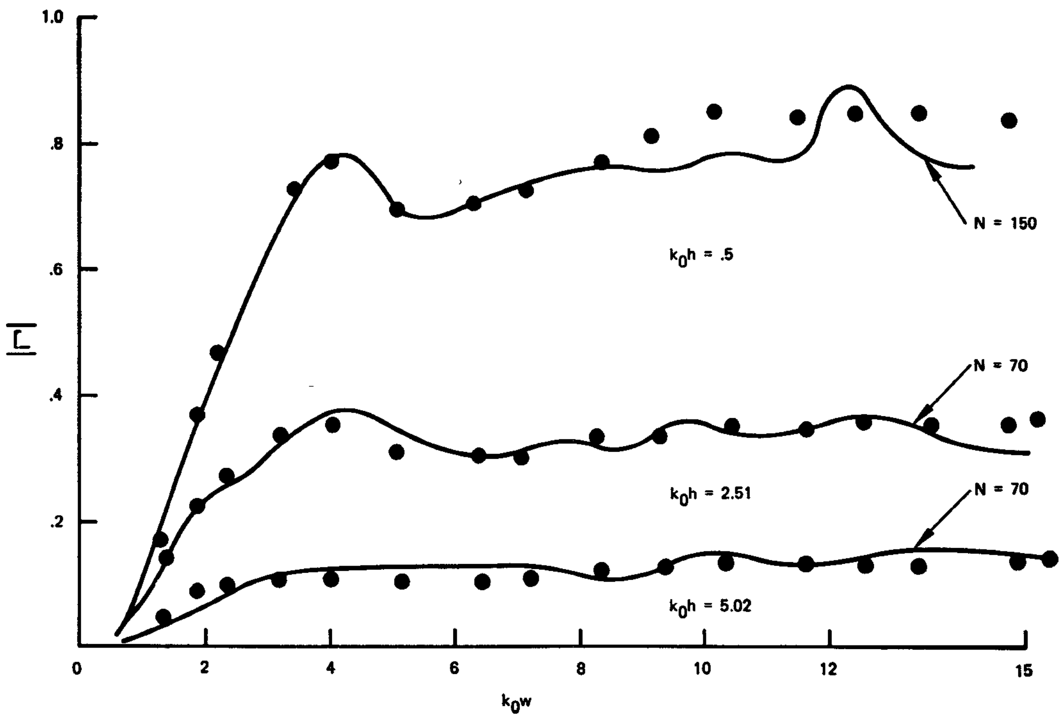

Fig. 2. Reflection coefficient versus strip width.

to present similar calculations for the case of strips over a dielectric-coated conducting plane.

To characterize the strips with a reasonable degree of completeness requires that the strips have a range of widths such that the asymptotic behavior can be determined. If the surface-wave parameters used in [3] are chosen, then strips with widths up to about three wavelengths are required. In the previous strip study data were presented for strips whose maximum widths were less than one wavelength; therefore, further calculations of the reflection coefficients for wider strips are required. As the strip width is increased the computational time increases quite rapidly. In [3] a variational formula for the reflection coefficient of the strip was developed, and the Rayleigh–Ritz technique was applied to obtain approximate values for the reflection coefficient. It was felt that some reduction in computer time might result if a more direct numerical approach were used; therefore, the problem is reformulated here.

II. Discussion of the Problem

The geometry of the problem is shown in Fig. 1. The strip is perfectly conducting and indefinitely thin and is located on arbitrary height above the dielectric-coated ground plane. The thickness of the dielectric is adjusted to permit only the lowest order TM surface-wave mode to propagate. As is well known the field components are proportional to exp $(-\alpha_0 y - j\beta_0 z)$ where α_0 is the attenuation constant for the y direction and β_0 is the propagation constant for the z direction.

The incident surface wave induces an electric current on the surface of the strip, which is only y directed. This current radiates a scattered field, part of which is backscattered as a surface wave, part is forward scattered also as a surface wave, and the remainder of the field is in the form of radiation away from the surface-wave system. The surface-wave components of the scattered field can be related to the incident field by reflection and transmission coefficients Γ and T, respectively, that is

$$(E_y^s)_{sw}^- \Big|_{z=0^-} = \Gamma E_y^i \tag{1}$$

and

$$(E_y^s)_{sw}^+ \Big|_{z=0^+} = T E_y^i \tag{2}$$

in which E_y^i represents the transverse component of the incident surface wave, $(E_y^s)_{sw}^-$ that of the reflected surface wave, and $(E_y^s)_{sw}^+$ that of the transmitted surface wave. Since the strip is assumed to be indefinitely thin its scattered field will be symmetrical about the $z=0$ plane. The total surface-wave field in the $z>0$ region being the

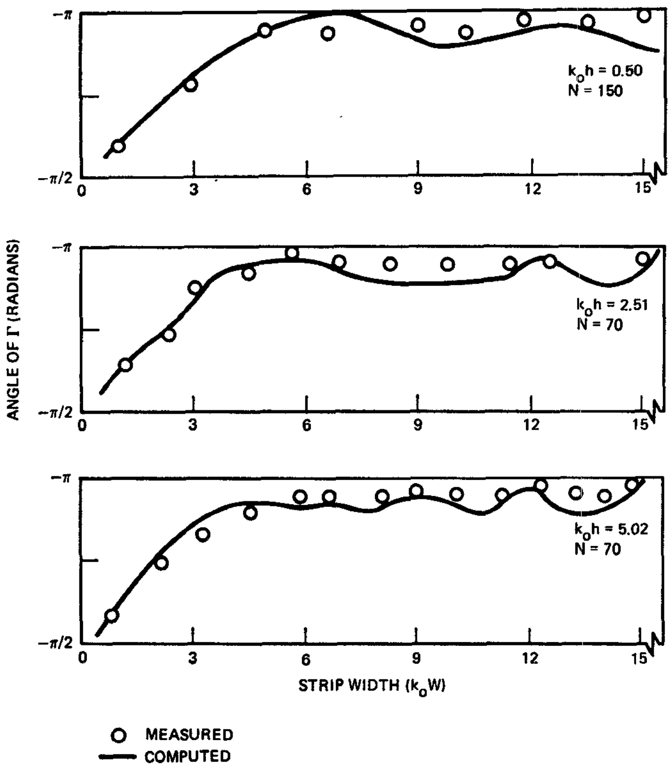

Fig. 3. The phase angle of the reflection coefficient versus strip width.

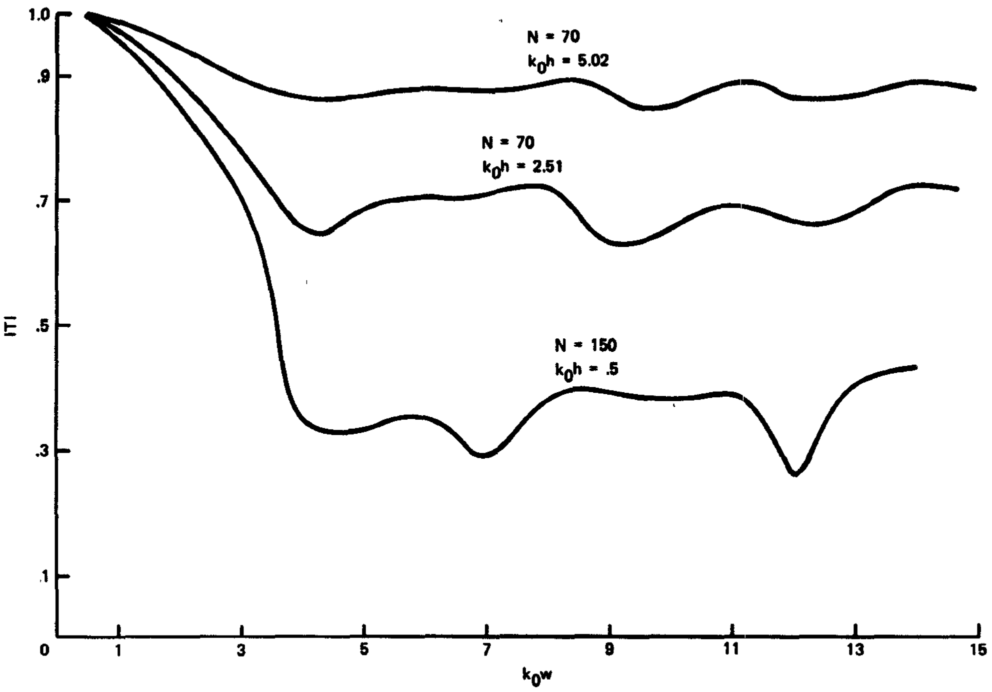

Fig. 4. Transmission coefficient versus strip width.

sum of the incident and scattered fields is given by

$$(E_y^s)_{sw}^+\Big|_{z=0^+} = (1+\Gamma)E_y^i.$$

A comparison with (2) reveals that

$$T = 1 + \Gamma. \tag{3}$$

The normalized impedance of the strip is given by [5]

$$\bar{Z} = -\frac{1+\Gamma}{2\Gamma} \tag{4}$$

where $\bar{Z}$ is the normalized shunt impedance. Finally, the fraction of power radiated is given by

$$S = 1 - \Gamma\Gamma^* - TT^* \tag{5}$$

in which $\Gamma\Gamma^*$, TT^*, and S represent the fraction of power reflected, transmitted, and radiated, respectively [5]. Thus, if the reflection coefficient is determined, that result can be used to compute the

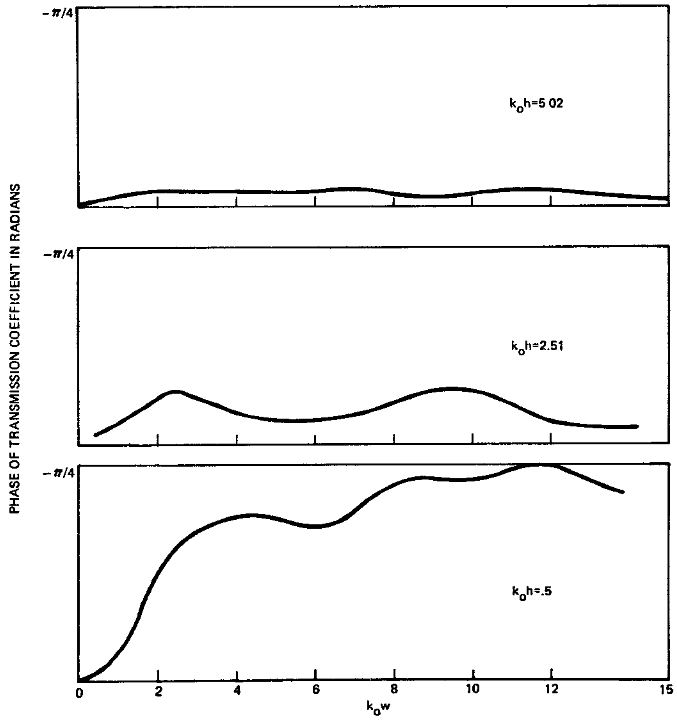

Fig. 5. Phase angle of transmission coefficient versus strip width.

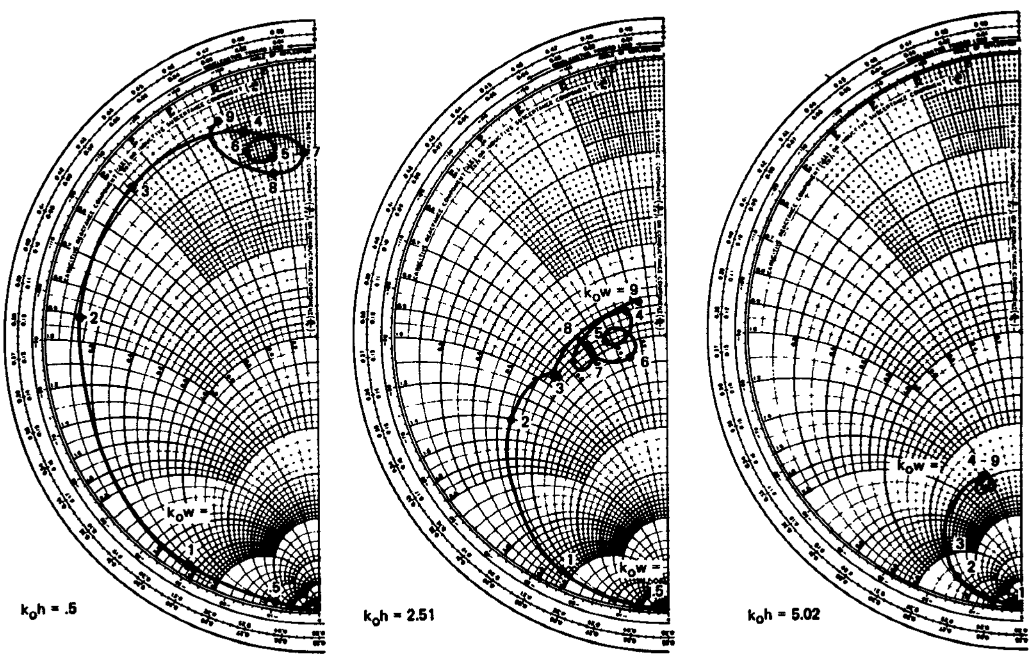

Fig. 6. Impedance versus strip width for $k_0h = 0.5, 2.51, 5.02$.

transmission coefficient, the shunt impedance, and the fractions of power reflected, transmitted, and radiated.

In [3] it was shown that the reflection coefficient for the strip can be calculated once the induced current $J(y')$ on the strip is known by use of

$$\Gamma = \frac{R_0\beta_0}{2\omega\epsilon_0}\int_{\text{STRIP}} e^{-\alpha_0 y'} J(y')\, dy \qquad (6)$$

where R_0 is given by

$$R_0 = \frac{-2\alpha_0}{1-\alpha_0[\kappa\alpha_0 + t(\kappa^2-1) + t(\kappa-1)k_0^2]/\kappa[(\kappa-1)k_0^2-\alpha_0^2]}$$

and t is the thickness of the dielectric, ω is the angular frequency of the incident surface wave, κ is the relative dielectric constant of the dielectric, and k_0 and ϵ_0 are the free space-wave number and permittivity, respectively.

The problem then is the determination of the unknown current $J(y')$. Once this is done the reflection coefficient can be obtained by use of (6); the transmission coefficient and shunt impedance are obtained through the use of (3) and (4).

III. Determination of the Induced Current

An appropriate integral equation from which the unknown current can be determined is given by [7]

$$-e^{-\alpha_0 y} + A' \sin k_0 y + B' \cos k_0 y = \int_{\text{STRIP}} G(y \mid y') J'(y')\, dy' \qquad (7)$$

in which the current has been scaled. The constants A and B are to be determined by the edge conditions; namely, that the currents vanish

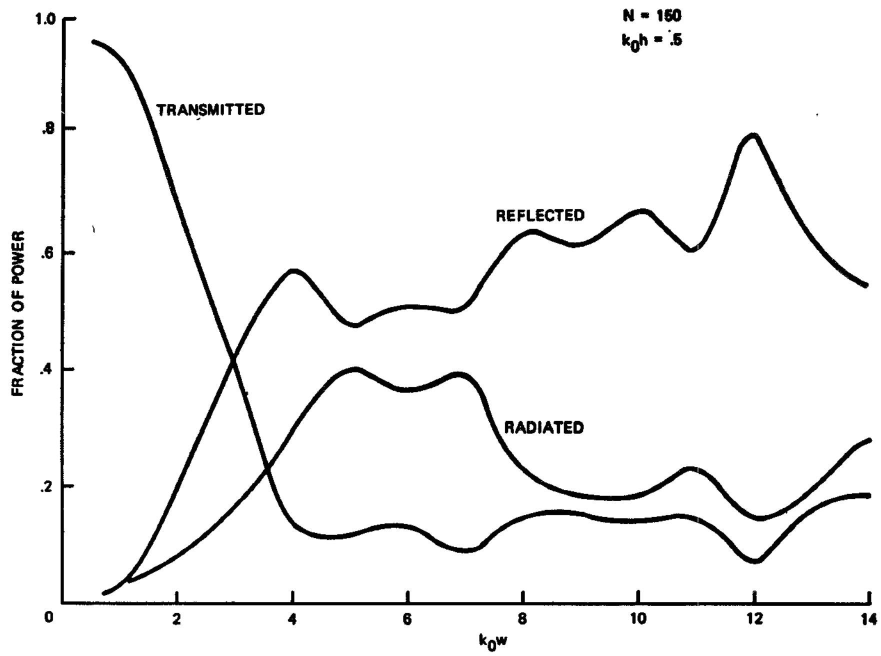

Fig. 7. Fraction of power radiated, reflected, and transmitted versus strip width for $k_0h = 0.5$.

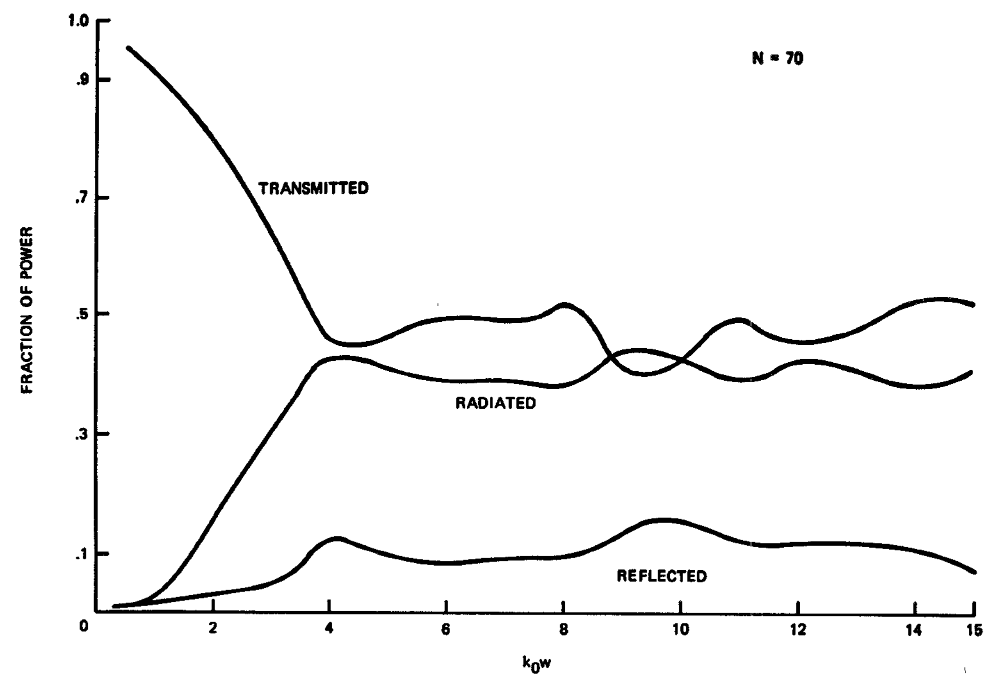

Fig. 8. Fraction of power radiated, reflected, and transmitted versus strip width for $k_0h = 2.51$.

at the top and bottom edge of the strip. Gustincic [3] has shown that for the plane $z=0$ the Green's function can be approximated by

$$G(y|y') \approx -j(\mu_0/4)\{H_0^{(2)}(k_0|y-y'|) + [(j\sqrt{\kappa}+\tan\sqrt{\kappa}\,k_0t)/(j\sqrt{\kappa}-\tan\sqrt{\kappa}\,k_0t)]H_0^{(2)}(k_0(y+y'))\} \quad (8)$$

in which $H_0^{(2)}(k_0|y-y'|)$ is the zeroth order Hankel function of the second kind, provided that the incident surface wave is lightly trapped; that is, $\sqrt{\kappa-1}\,k_0t \ll (\pi/2)$ where t is the thickness of the dielectric. This approximation has been shown [6] to be highly accurate for heights such that $k_0h > 0.5$. The integral contained in (7) can be approximated by a summation. This is accomplished by dividing the strip into N equal intervals. This procedure yields a system of N equations, of which the jth one is given by

$$-E_j + A'S_j + B'C_j = \Delta \sum_{i=1}^{N} G_{ji}J_i' \quad (9)$$

where Δ is the width of the interval, the j index is associated with the field position y, and i with the source position y'. In (9) E_j, S_j, and C_j represent the exponential function, sine function, and cosine function, respectively.

There is the additional requirement that the current be zero at the edges of the strip. This condition can be imposed by properly choosing A' and B'. To do this, set J_1 and J_N equal to zero, then let $j=1$ and $j=N$. A' and B' can be determined by the simultaneous solution of these two equations. When these results are substituted into (9) the jth equation of the system of equations can be written as

$$D\{(E_1C_N - E_NC_1)S_j - (E_1S_N - E_NS_1)C_j\} - E_j = \Delta \sum_{i=2}^{N-1} \{G_{ji} - D[(C_NG_{1i} - C_1G_{Ni})S_j - (S_NG_{1i} - S_1G_{Ni})C_j]\}J_i' \quad (10)$$

where the $i=1$, N terms have been dropped since $J_1 = j_N = 0$.

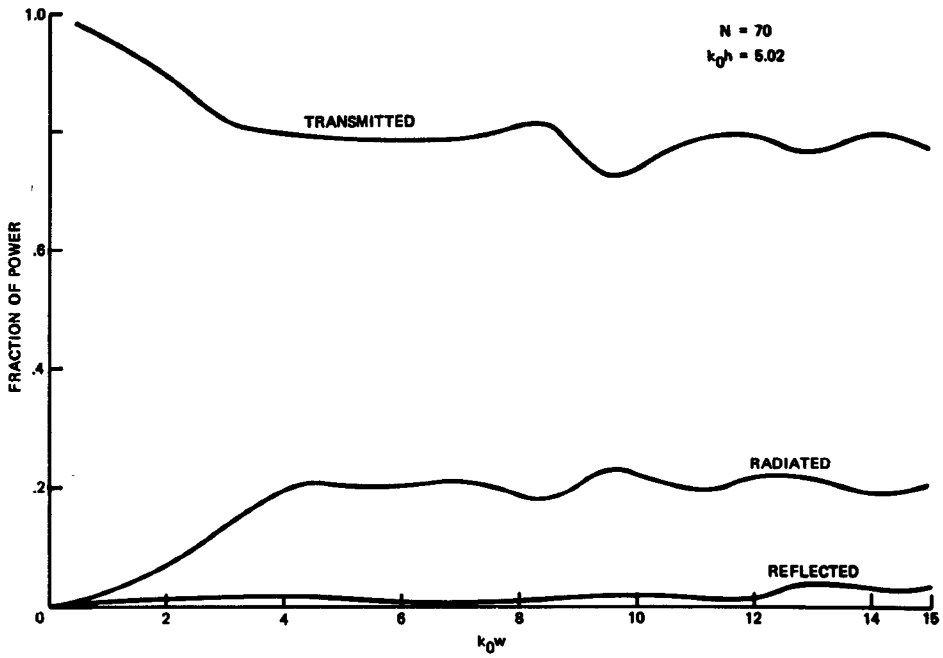

Fig. 9. Fraction of power radiated, reflected, and transmitted for $k_0h = 5.02$.

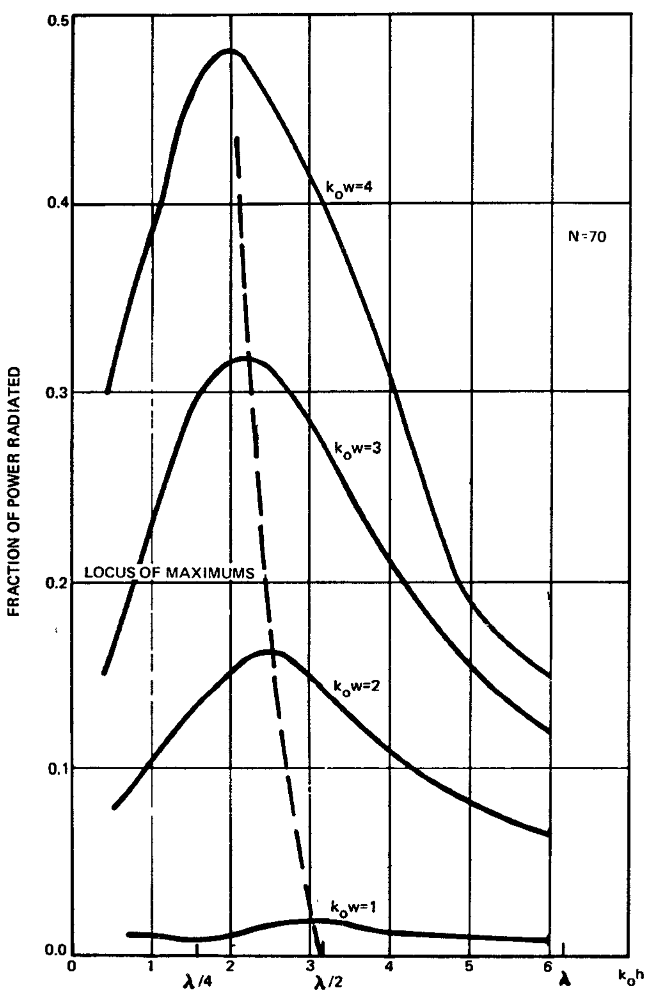

Fig. 10. Fraction of power radiated versus height for strips with constant widths.

One difficulty remains, namely, the singularity that occurs when $y=y'$. It is the Hankel function $H_0^{(2)}(k_0|y-y'|)$ that contains the singularity. The procedure used is as follows. In the vicinity of $y=y'$, the Hankel function is replaced by its small argument form which is integrable, and its average value over the interval is taken. This procedure yields

$$\overline{H_0^{(2)}(k_0|y-y'|)} \approx 1 - j\frac{2}{\pi}\left\{\ln\frac{\gamma}{2} - 1 + \ln\frac{\Delta}{2}\right\} \quad (11)$$

in which γ is Euler's constant and the bar represents "the average value of." The unknown current distribution can now be determined by the inversion of (10). Whenever $i=j$, (11) is used instead of the Hankel function. The Gauss–Jordon technique was used to solve this system of equations.

IV. Results

The parameters of the surface-wave system used for the computation and experiment are: the thickness of the dielectric, $k_0t=0.312$; attenuation constant, $(\alpha_0/k_0)=0.188$; and the dielectric constant, $\kappa=2.54$. The experimental apparatus was designed to operate at 9.375 GHz. Typical computed and measured values of the magnitude and phase angle of the reflection coefficients for strips of various widths and heights above the surface-wave system are displayed graphically in Figs. 2–3. The experimental values were obtained from measurements made with the use of the apparatus described in detail in [3].

An examination of the computed values shown in Fig. 2 reveals, not surprisingly, that for a given strip width the closer the strip is to the dielectric surface the more increments that are required for the numerical solution. For example, notice that for $k_0h=2.51$ and $k_0h=5.02$ the strip was divided into 70 increments. Excellent correspondence between calculated and measured values for strip widths over the full range of values considered was obtained when $k_0h=5.02$. Note, however, that when the height is lowered to $k_0h=2.51$ a significant deviation from the experimental values is noted when $k_0w>13.5$. Although not shown, the calculated data began to deviate from the measured data for $k_0h=0.5$ when k_0w was only equal to 3. When N was increased to 150 the correspondence was good up to $k_0w=8$, as can be seen from Fig. 2. Even so, in the latter case the deviation from the experiment is only eight percent for widths up to $k_0w=15$.

The transmission coefficient can be computed with the use of (3). The results are shown in Figs. 4 and 5. The normalized impedance is computed with the use of (4). These results are shown in Fig. 6. Note that the real part of the impedance for each case considered remains very nearly constant as the strip width is varied. Furthermore, once the width reaches a certain value the impedance stays fairly constant thereafter, with the impedance curve forming loops about a constant value. Also, the impedance is always capacitive as one might expect.

The computed fractions of power reflected, radiated, and transmitted are displayed in Figs. 7–9. Notice that as a strip of a given width is moved away from the surface, that is, as k_0h increases, the fraction of reflected power decreases and that of the transmitted power increases, both in a seemingly monotonic fashion. The fraction of radiated power, on the other hand, first increases then decreases, giving rise to a pronounced maximum at a critical height. This was investigated by computing the fraction of power radiated as a function of height with the width as a parameter. The results are shown in Fig. 10. Sketched in that figure is the locus of the maxima. Note that for narrow strips the maxima occur at a height very nearly one-half wavelength, whereas for wide strips the maxima occur at heights that are closer to one-quarter wavelength. Note the smooth transition in the heights where the maxima occur as the strip widths vary from narrow to wide.

Another striking feature of the results is that the maxima never exceed 0.5, although they do tend to approach that value for wider strips. While no formal proof is given it does appear that the maximum fraction of radiated power of a conducting strip might be 0.5.

Acknowledgment

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Experimental Gain and Noise Parameters of Microwave GaAs FET's in the L and S Bands

A. ANASTASSIOU AND M. J. O. STRUTT

***Abstract*—The design of microwave amplifiers with GaAs FET's assumes the knowledge of the four gain and the four noise parameters as a function of the biasing conditions. The gain parameters at three different bias conditions have been calculated by computer from the measured scattering parameters. The noise figures as a function of the same bias conditions have also been measured. The four fundamental noise parameters have been determined. The GaAs FET's are units from Plessey (England). At present, these are the only units which are commercially available.**

I. Gain Parameters

The available gain of a two-port as a function of the four gain parameters is given by

$$\frac{1}{G_{av}} = \frac{1}{G_{avo}} + \frac{Q_g}{1 - U_s^2 - V_s^2}[(U_s - U_{og})^2 + (V_s - V_{og})^2] \quad (1)$$

where G_{avo} is the maximum available gain of the two-port, $r_{og}=U_{og}+jV_{og}$ the optimum complex source reflection coefficient with respect to gain, Q_g a factor which indicates the dependence of the available gain on the complex source reflection coefficient, and $r_s=U_s+jV_s$ the complex reflection coefficient of the source.

The four gain parameters as a function of the scattering parameters, which are determined in [2], are given by

$$G_{avo} = \frac{|s_{21}|}{|s_{12}|}\cdot(K - \sqrt{K^2-1}) \quad (2)$$

$$Q_g = \frac{1}{G_{avo}} + \frac{|s_{11}|^2 - |\Delta|^2}{|s_{21}|^2}, \qquad \Delta = s_{11}s_{22} - s_{21}s_{12} \quad (3)$$

$$U_{og} = \frac{\mathrm{Re}\,(C)}{|s_{21}|^2\cdot Q_g}, \qquad C = s_{11} = s_{22}^*\,\Delta \quad (4)$$

$$V_{og} = \frac{-\mathrm{Im}\,(C)}{|s_{21}|^2\cdot Q_g}. \quad (5)$$

Here, K is the stability factor. If $K>1$, the two-port is unconditionally stable and the gain parameters are given by (2)–(5). If $K<1$, the two-port is conditionally stable and the available gain becomes equal to the maximum stable gain (MSG):

$$\mathrm{MSG} = \frac{|s_{21}|}{|s_{12}|}. \quad (6)$$

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